Bayesian inference in EEG/MEG

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"Will it ever happen that mathematicians will know enough about the physiology of the brain, and neurophysiologists enough of mathematical discovery, for efficient cooperation to be possible"

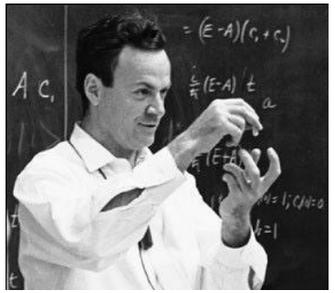
Jacques Hadamard *(mathematician, 1865-1963)*

Workshop on Bayesian approaches in neuroscience – Karolinska Institute – 2nd of Frebruary 2016

Previously…

"What I cannot create, I cannot understand"

Richard Feynman *(Nobel laureate in Physics, 1918-1988)*



- Bayesian or Probabilistic modelling: accounts for knowledge/uncertainty in the process of interest
- Generative or phenomenological models

<u>Ex. Dynamic causal models (DCM)</u>

Hidden states

Unknown parameters

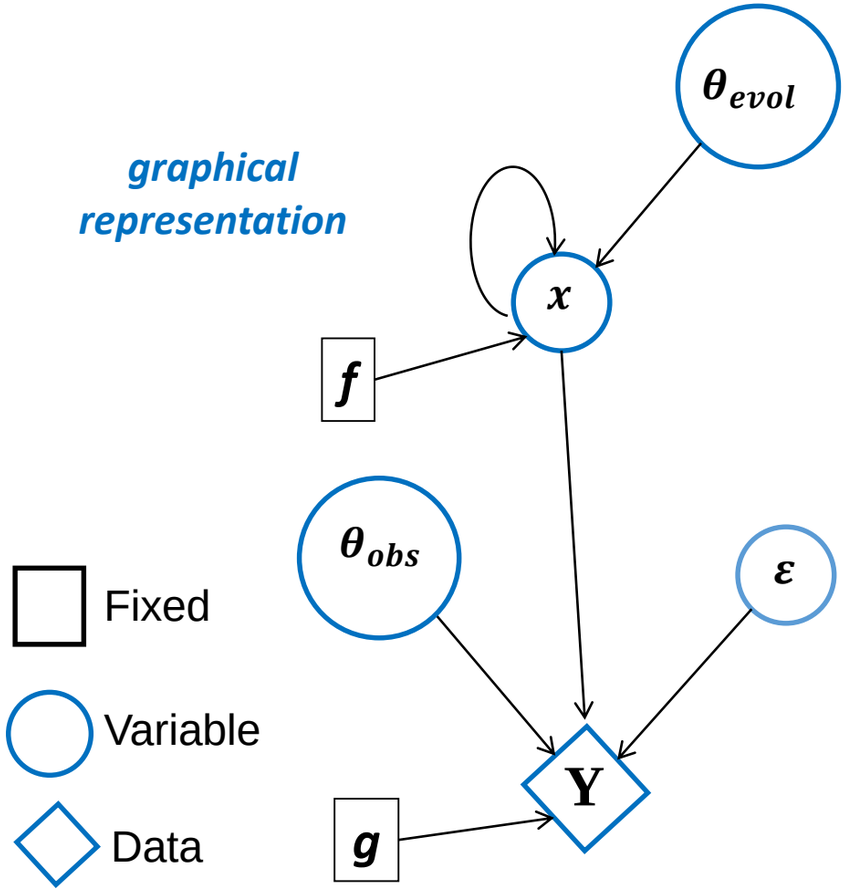
evolution $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{\theta}_{evol}, \boldsymbol{u})$

observation $\boldsymbol{y} = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{\theta}_{obs}) + \boldsymbol{\varepsilon}$

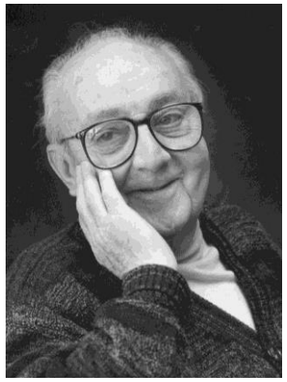
Inputs

Observations (data features)

graphical representation



Previously…



"All models are wrong but some are useful"

George E. P. Box *(statistician, 1919-2013)*

- Bayesian or Probabilistic modelling: accounts for knowledge/uncertainty in the process of interest
- Generative or phenomenological models
- Inference on models

Model M : your scientific hypothesis put into predictive equations (functions *f* and *g* + prior assumptions)

Likelihood

priors

$$P(\theta|Y,M)=\frac{P(Y|\theta,M)P(\theta|M)}{\boxed{P(Y|M)}}$$

Evidence

- Usually approximated (FE, AIC, BIC)
- Trades accuracy and complexity
- Model comparison through Bayes Factors and model posteriors
- Models can only be compared on the same data

Previously…

- Bayesian or Probabilistic modelling: accounts for knowledge/uncertainty in the process of interest
- Generative or phenomenological models
- Inference on models
- Inference on model parameters

Most plausible model or model family

$$\boxed{P(\theta|Y,M)} = \frac{P(Y|\theta,M)P(\theta|M)}{P(Y|M)}$$

Posterior

- Inference at the group level

Outline

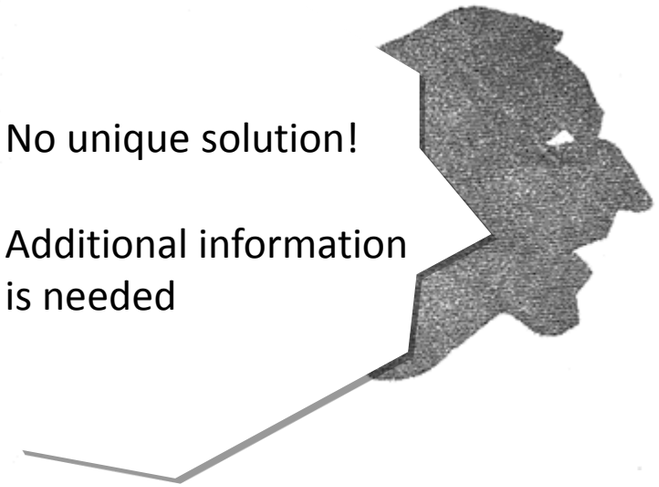
- **1 - EEG/MEG Source reconstruction**
- **2 - Inferring learning mechanisms from EEG/MEG responses**
- **3 - Online Bayesian inference and design optimization**

1 – EEG/MEG source reconstruction

An ill-posed inverse problem

No unique solution!

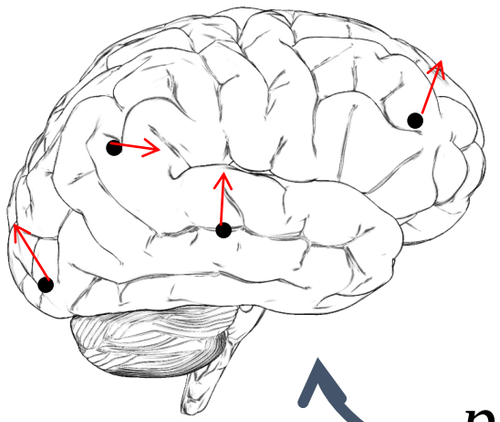
Additional information is needed



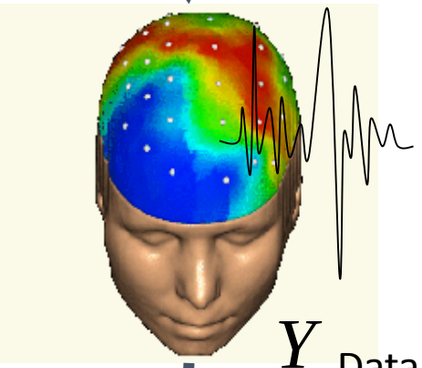
Generative model M

$p(Y \mid \theta, M)$ Likelihood

$p(\theta \mid M)$ Prior



θ Parameters



Y Data

Posterior $p(\theta \mid Y, M)$

Evidence $p(Y \mid M)$

Inverse Problem

Bayesian inference

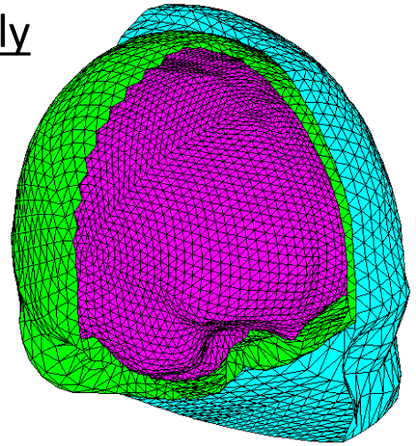
1 – EEG/MEG source reconstruction

Typical model families

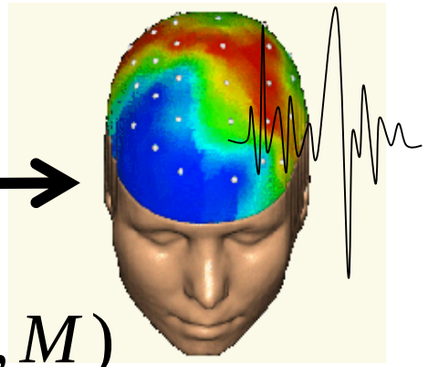
- **Biophysical model**

Static (forward) model family

Geometrical and physical properties of the head tissues



$p(Y \mid \theta, M)$

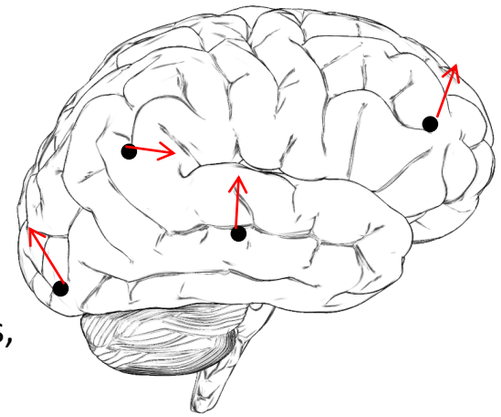


Y: sensor data (incl. noise)

- **Source model**

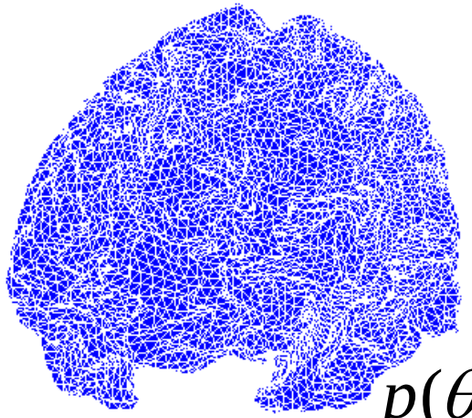
Equivalent current dipole (ECD) model family

θ : dipole locations, orientations & intensity



$p(\theta \mid M)$: number of dipoles, initial location

Distributed sources or Imaging model family

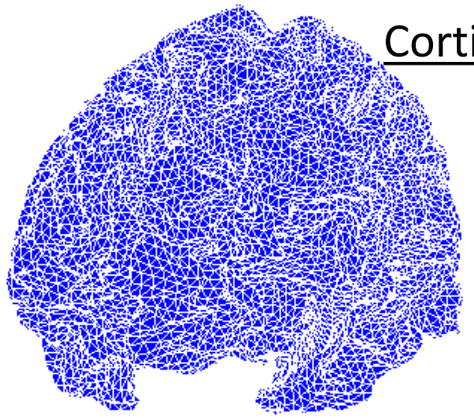


θ : dipole intensities

$p(\theta \mid M)$: number of dipoles, fixed locations

1 – EEG/MEG source reconstruction

Detailed example: the distributed source model

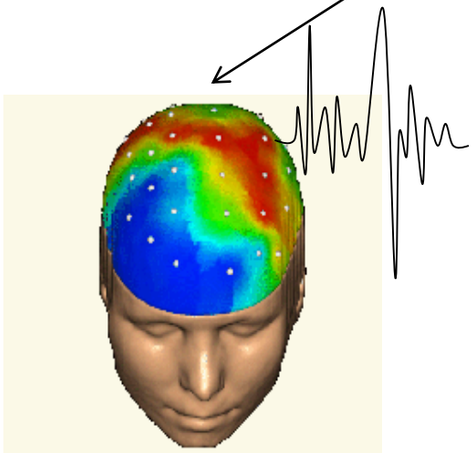


Cortical mesh (3D surface)

θ : dipole intensities

$$\boldsymbol{Y} = \boldsymbol{K}.\boldsymbol{\theta} + \boldsymbol{\varepsilon}$$

Additive noise



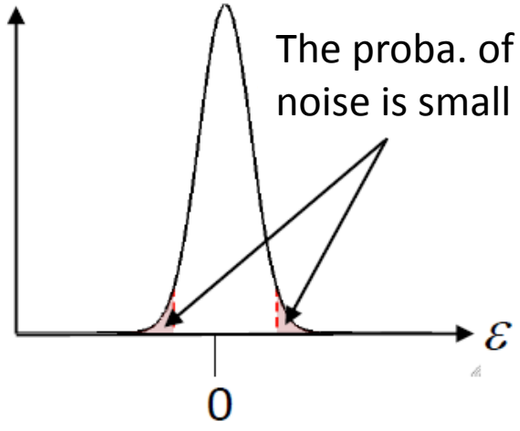
Unknown source dynamics: Nsources x Time

Forward operator or lead-field matrix: Nsens x Nsources

Evoked response: Nsens x Time

- **Likelihood**

$$\boldsymbol{p}(\boldsymbol{\varepsilon}|\boldsymbol{M}) = \boldsymbol{N}(\boldsymbol{0}, \boldsymbol{C}_{\boldsymbol{\varepsilon}})$$



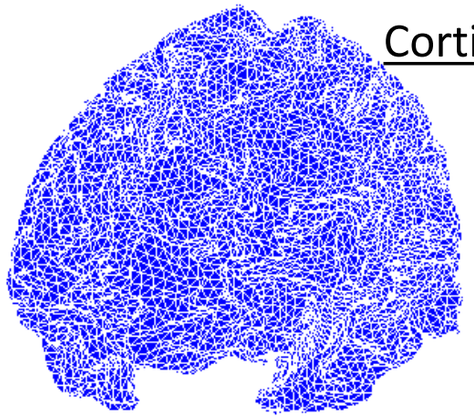
$$\boldsymbol{p}(\boldsymbol{Y}|\boldsymbol{\theta}, \boldsymbol{M}) = \boldsymbol{N}(\boldsymbol{K}.\boldsymbol{\theta}, \boldsymbol{C}_{\boldsymbol{\varepsilon}})$$

- **Prior**

$$\boldsymbol{p}(\boldsymbol{\theta}|\boldsymbol{M}) = \boldsymbol{N}(\boldsymbol{0}, \boldsymbol{C}_{\boldsymbol{\theta}})$$

1 – EEG/MEG source reconstruction

Detailed example: the distributed source model



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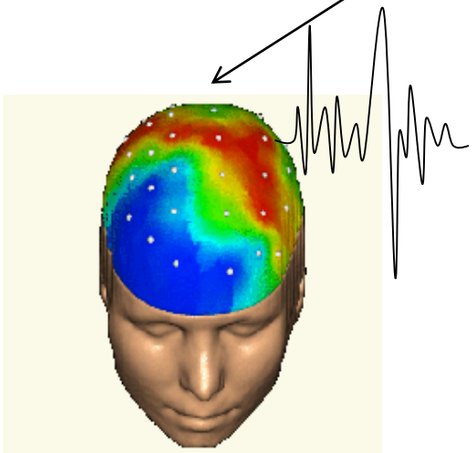

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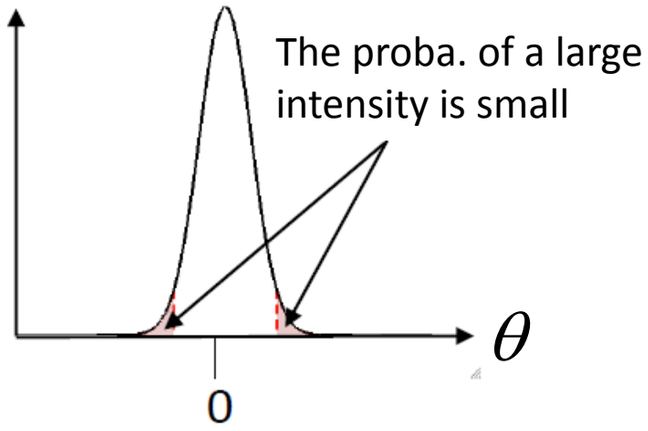


- **Prior**

$$\boldsymbol{p}(\boldsymbol{\theta}|\boldsymbol{M}) = \boldsymbol{N}(\boldsymbol{0}, \boldsymbol{C}_{\boldsymbol{\theta}})$$

1 – EEG/MEG source reconstruction

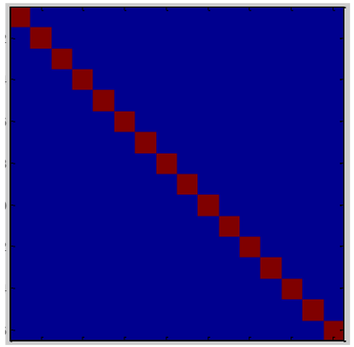
Philips et al. 2005
Mattout et al. 2005 & 2006
Wipf and Nagarajan 2009

Commonly used priors

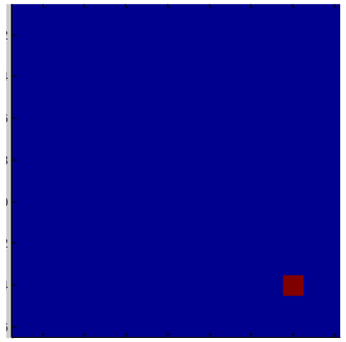
- **Priors**

$$p(\theta|M) = N(0, C_\theta)$$

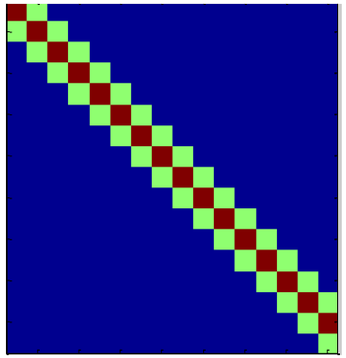
Nsources x Nsources



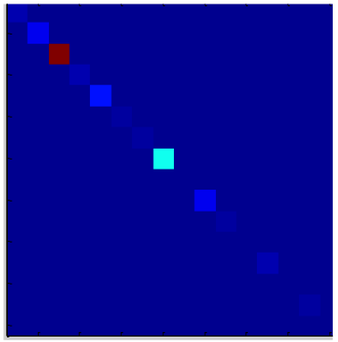
i.i.d or Minimum norm



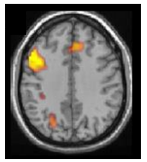
Single dipole

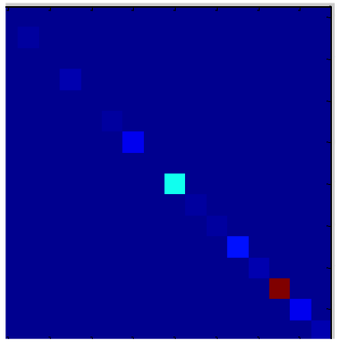


Smoothness (like LORETA)



fMRI based



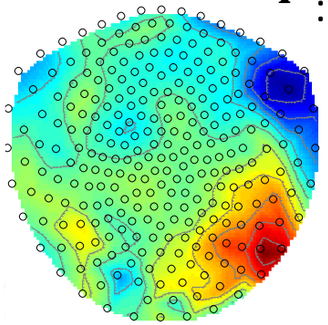


Lead-field based
(Beamformer or MSP)

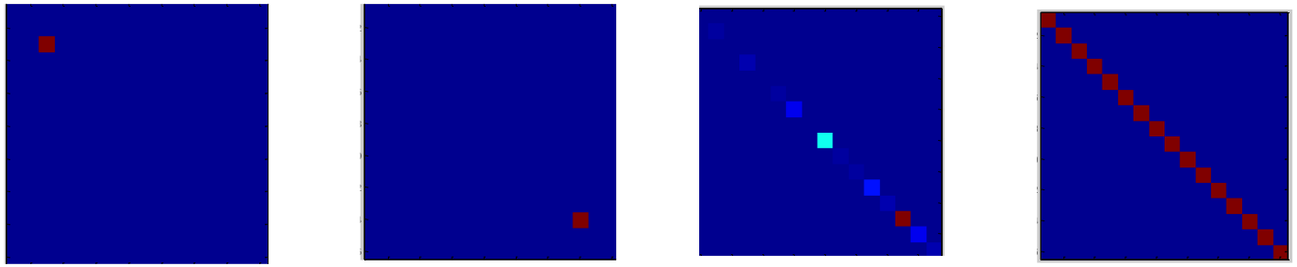
1 – EEG/MEG source reconstruction

Comparing priors

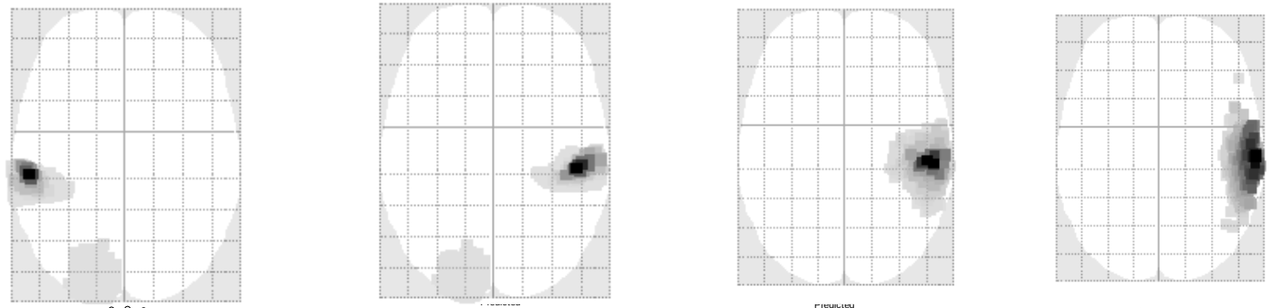
$\boldsymbol{Y}$: observed data



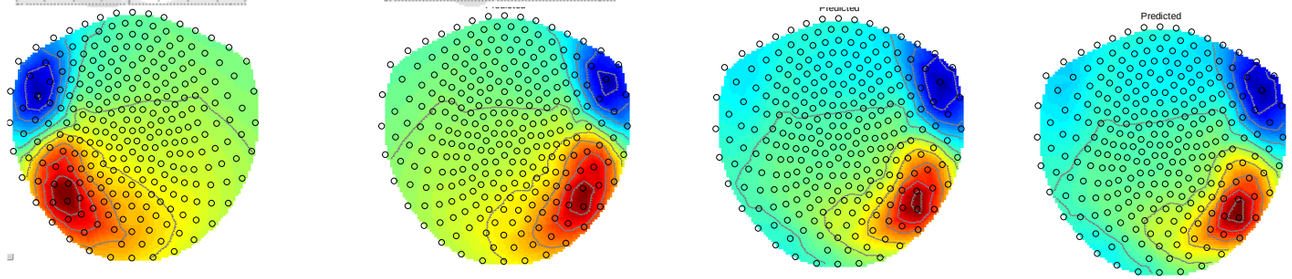
Alternative priors



$\hat{\boldsymbol{\theta}}$: estimated source intensities



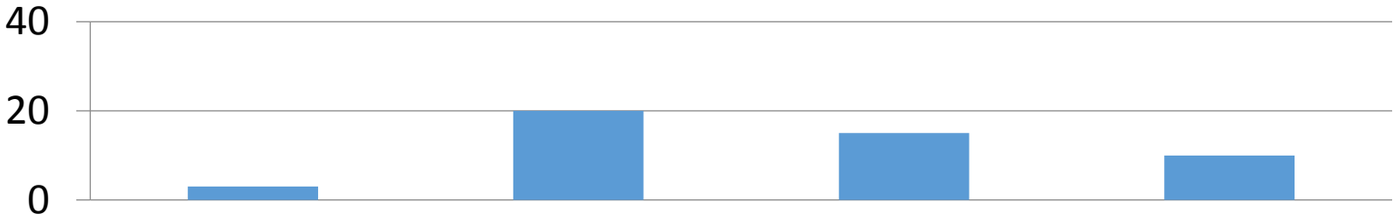
$\hat{\boldsymbol{Y}}$: predicted data



Explained variance

11 % 96% 97% 98%

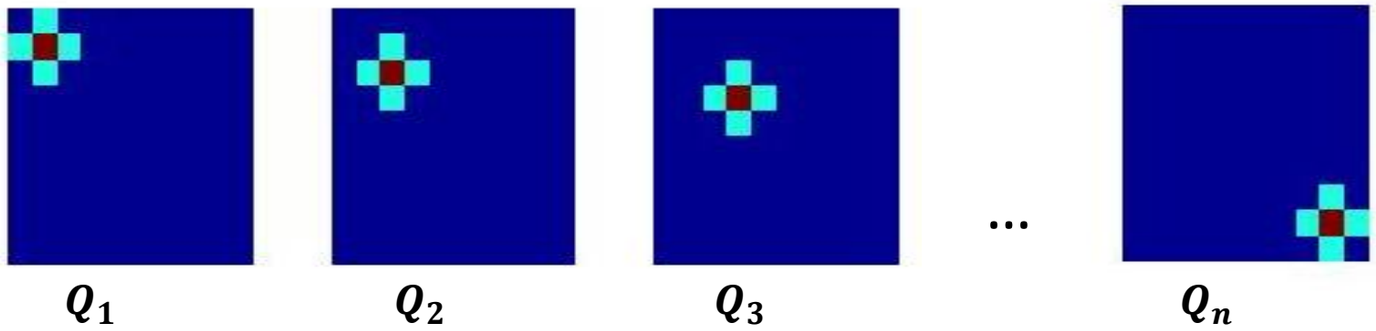
Log-evidence



1 – EEG/MEG source reconstruction

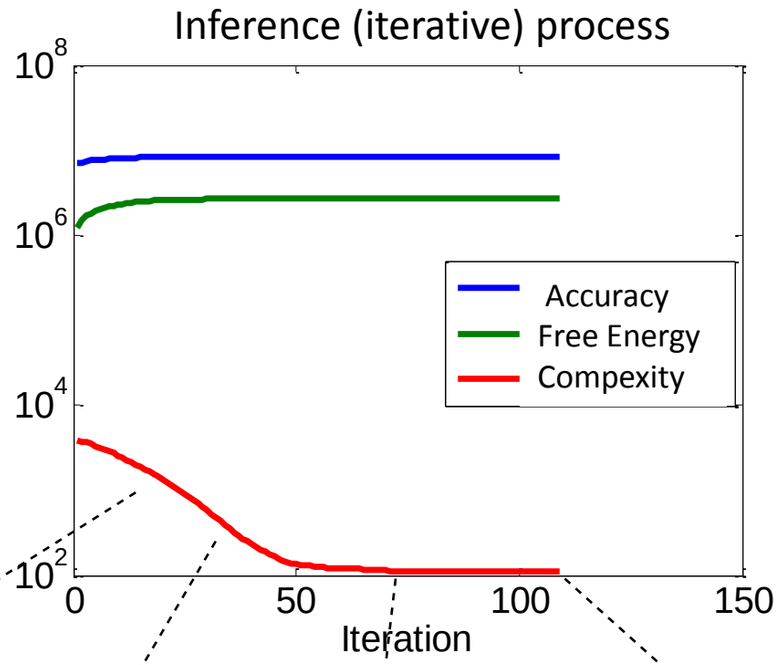
Empirical Bayes: multiple sparse priors (MSP)

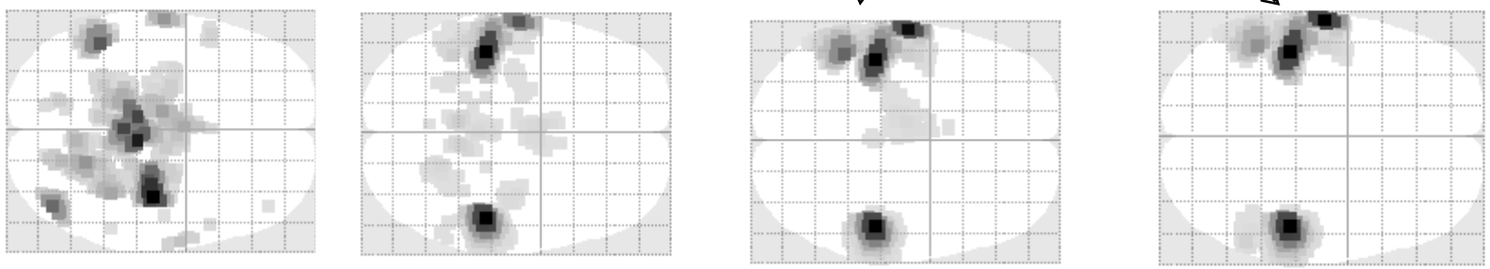
Friston et al. 2008



Model (Hyper)parameters

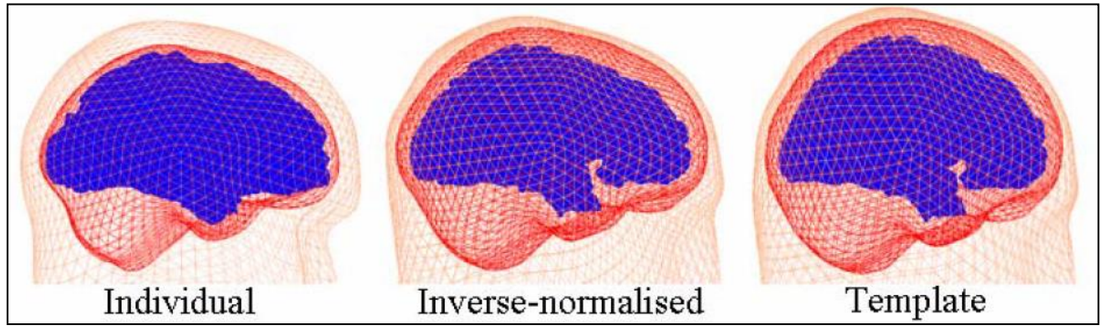
$$C_\theta = \lambda_1 . Q_1 + \lambda_2 . Q_2 + \lambda_3 . Q_3 + \cdots + \lambda_n . Q_n$$





1 – EEG/MEG source reconstruction

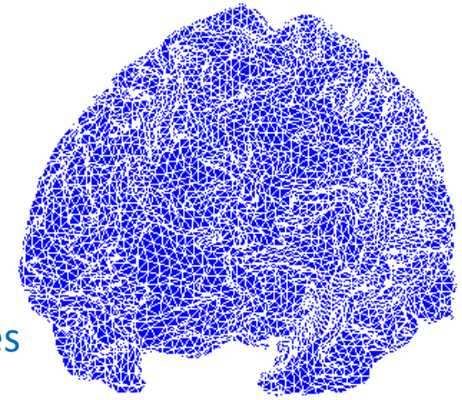
Model comparison on other dimensions



Anatomy

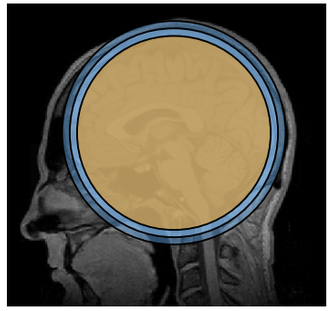
Mattout et al. 2007

Sources

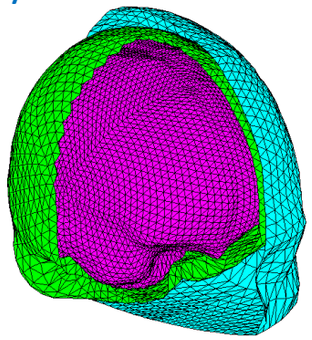


Mesh resolution (High / Low)
Dipole oreintation (fixed / free)

Biophysics



Spherical head model



Realistic surfacic model (BEM)

Model comparison on real MEG data →
Henson et al. 2007

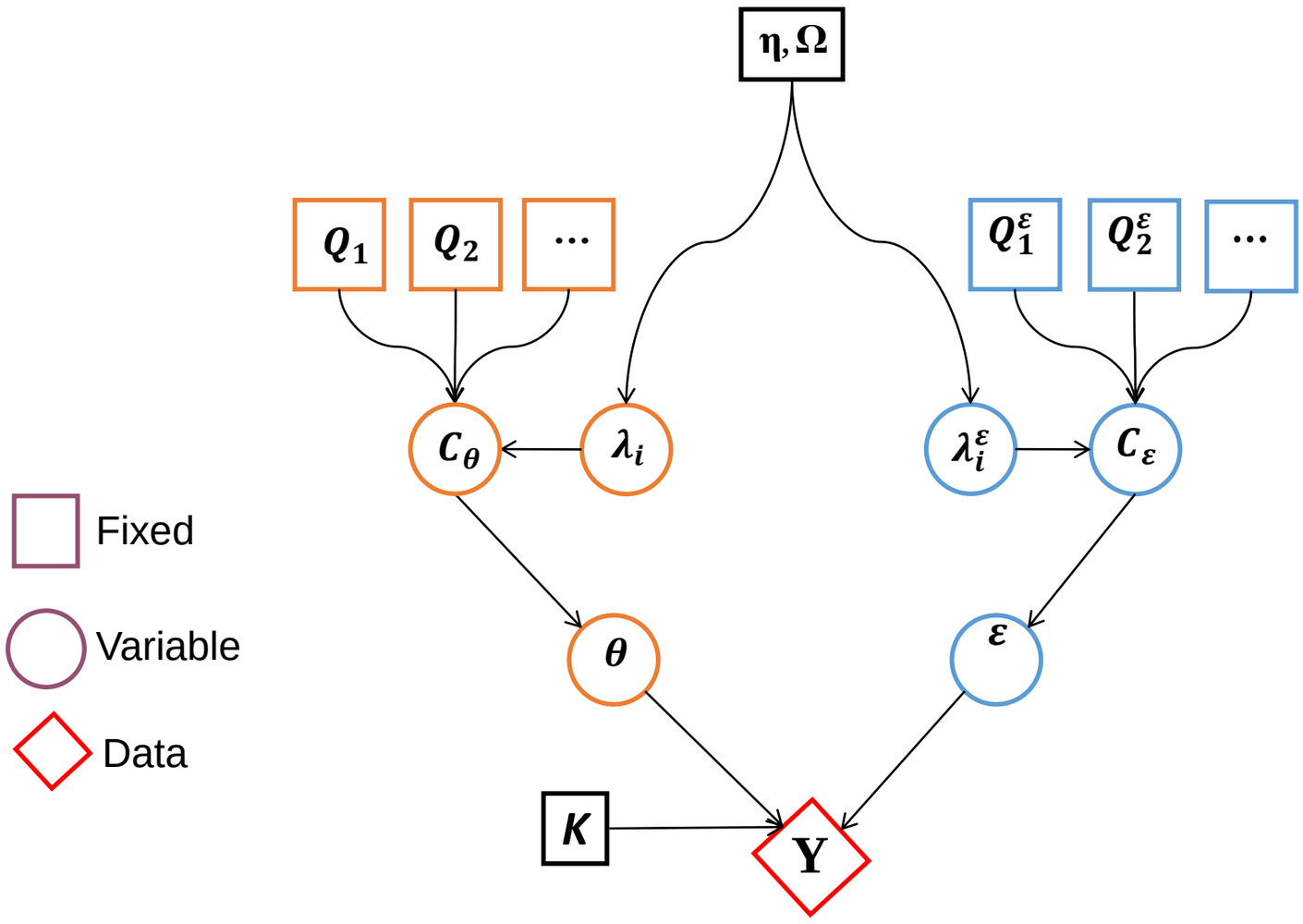
- **No evidence in favor of individual vs. Inverse-norm mesh**
- **Evidence in favor of BEM head model**
- **Evidence in favor of high + fixed vs. low + free**

Henson et al. 2009

1 – EEG/MEG source reconstruction

Graphical representation of MSP

Directed Acyclic Graph (DAG) representation



Friston et al. 2008

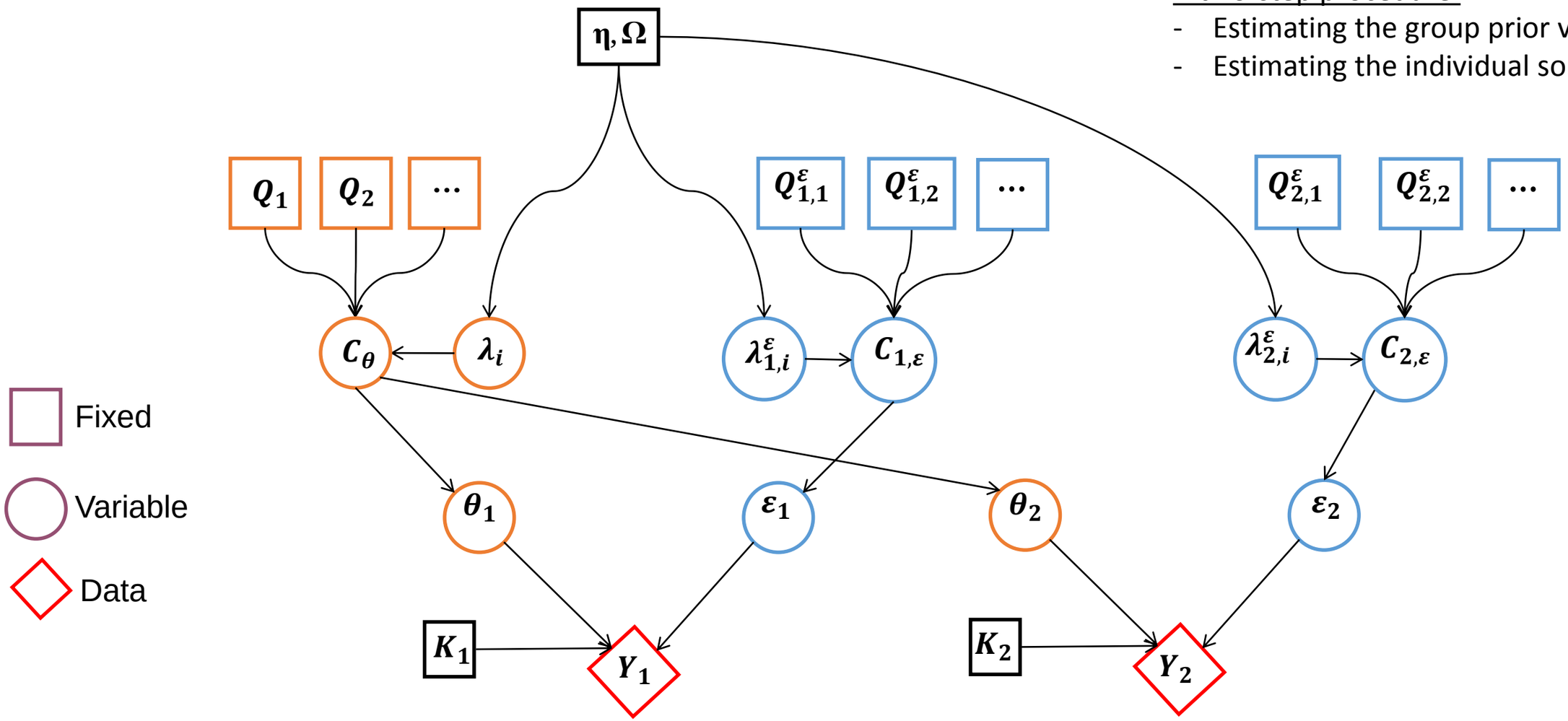
1 – EEG/MEG source reconstruction

Model extension to group level inference

Directed Acyclic Graph (DAG) representation

A two-step procedure:

- Estimating the group prior variance
- Estimating the individual source intensities

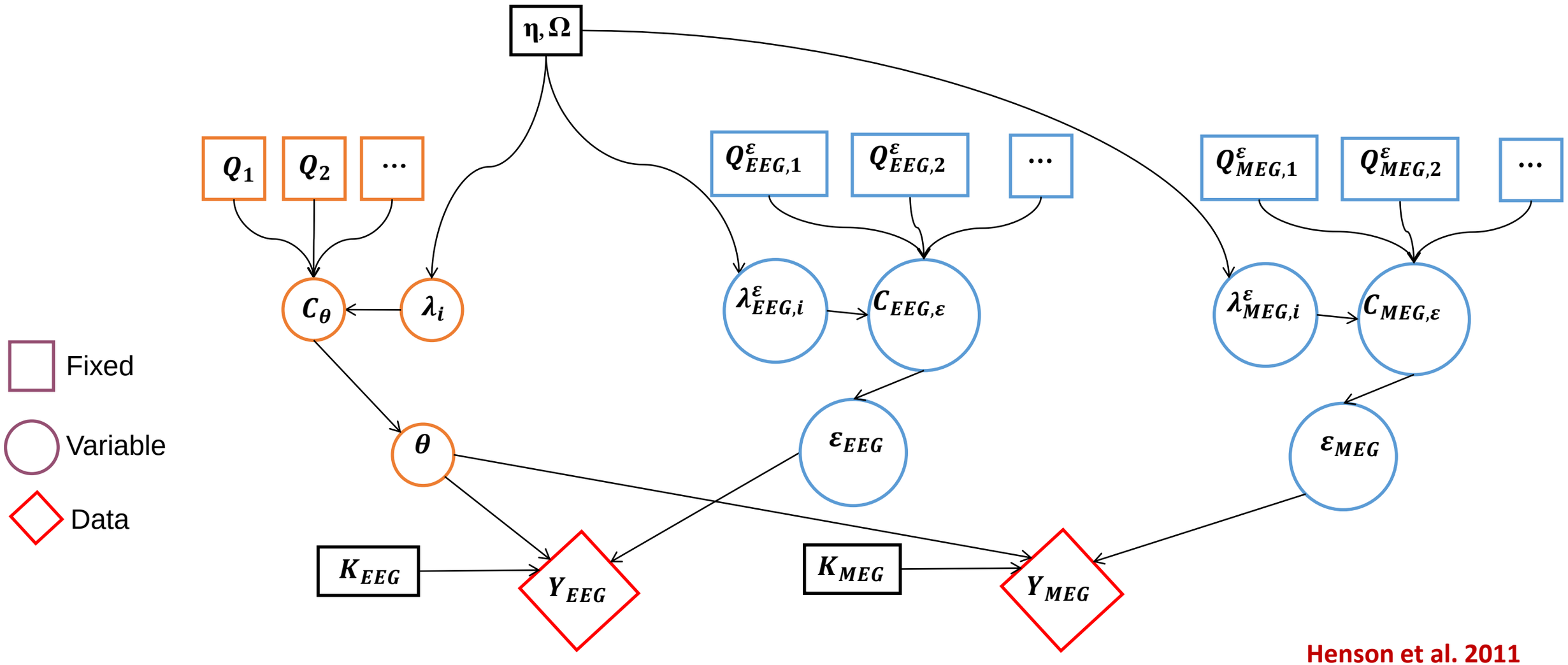


Litvak and Friston 2008

1 – EEG/MEG source reconstruction

Model extension to EEG-MEG data fusion

Directed Acyclic Graph (DAG) representation



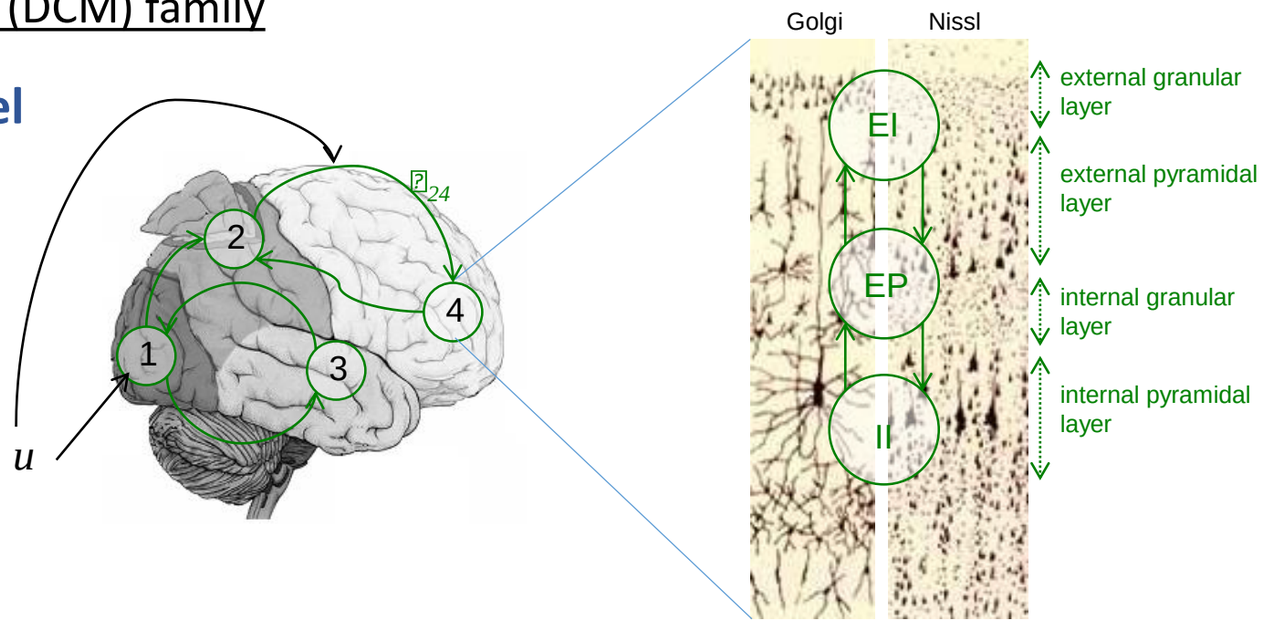
Henson et al. 2011

1 – EEG/MEG source reconstruction

Another model family

Dynamic causal model (DCM) family

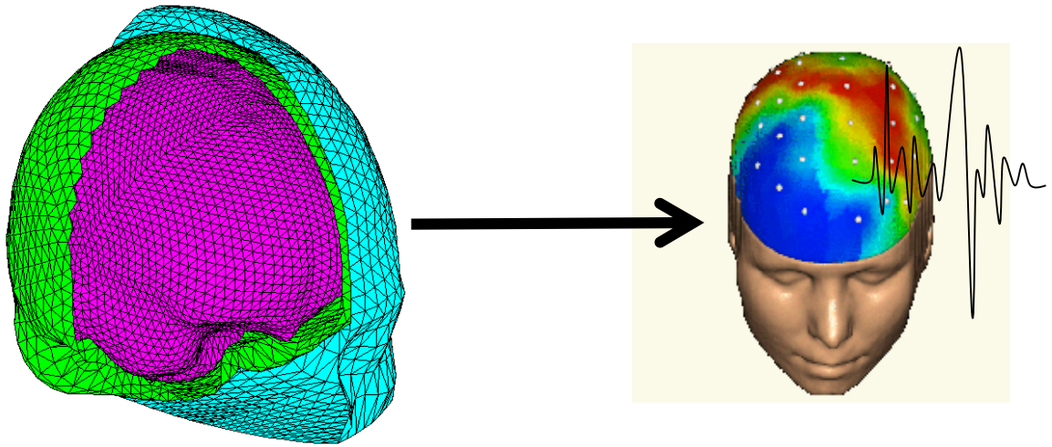
- **Source model**



$$\dot{x} = f(x, \theta_{evol}, u)$$

Model parameters:
- Source intrinsic parameters (e.g. synaptic gain)
- Extrinsic coupling
- Source orientation

- **Biophysical model**



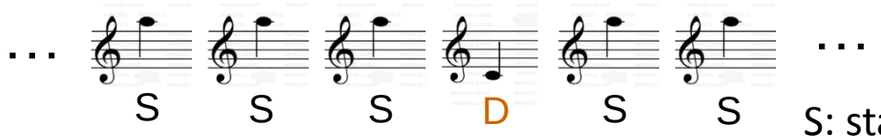
$$y = g(x, \theta_{obs}) + \varepsilon$$

1 – EEG/MEG source reconstruction

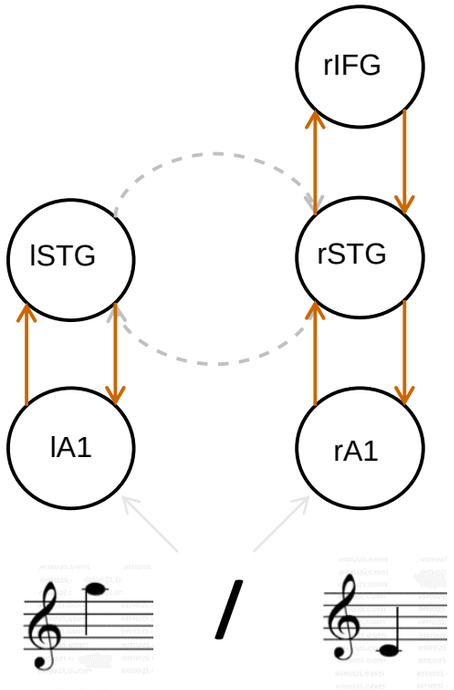
An MMN example

Characterizing a group response in terms of modulations of effective connectivity

Classical auditory oddball paradigm



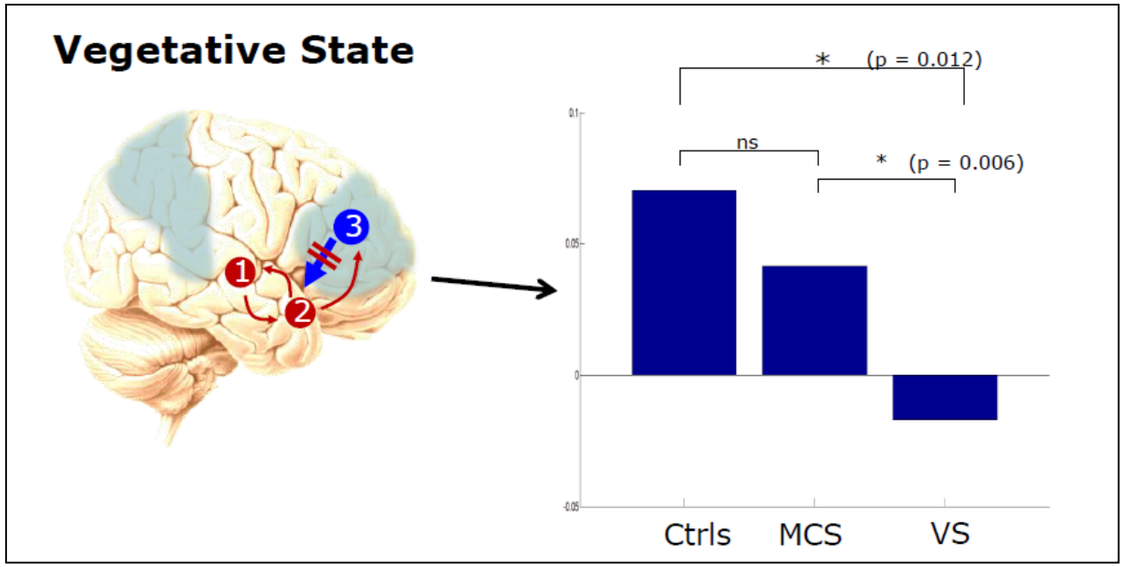
S: standard tone
D: (frequency) deviant tone



Garrido et al. 2009

Ctrls: Controls
MCS: Minimally Conscious State
VS: Vegetative State

Boly et al. 2011



2 – Inferring learning mechanisms from EEG/MEG data

2 – Inferring learning mechanisms from EEG/MEG data

Neurophysiological correlates of implicit perceptual learning

- **In line with the Bayesian brain hypothesis and its neuronal instantiation (predictive coding), it has been suggested that evoked responses reflect prediction error**

Friston 2005

- **Deviance responses should be modulated by predictability**

- **Single trial modulations of evoked responses should reflect perceptual learning**

2 – Inferring learning mechanisms from EEG/MEG data

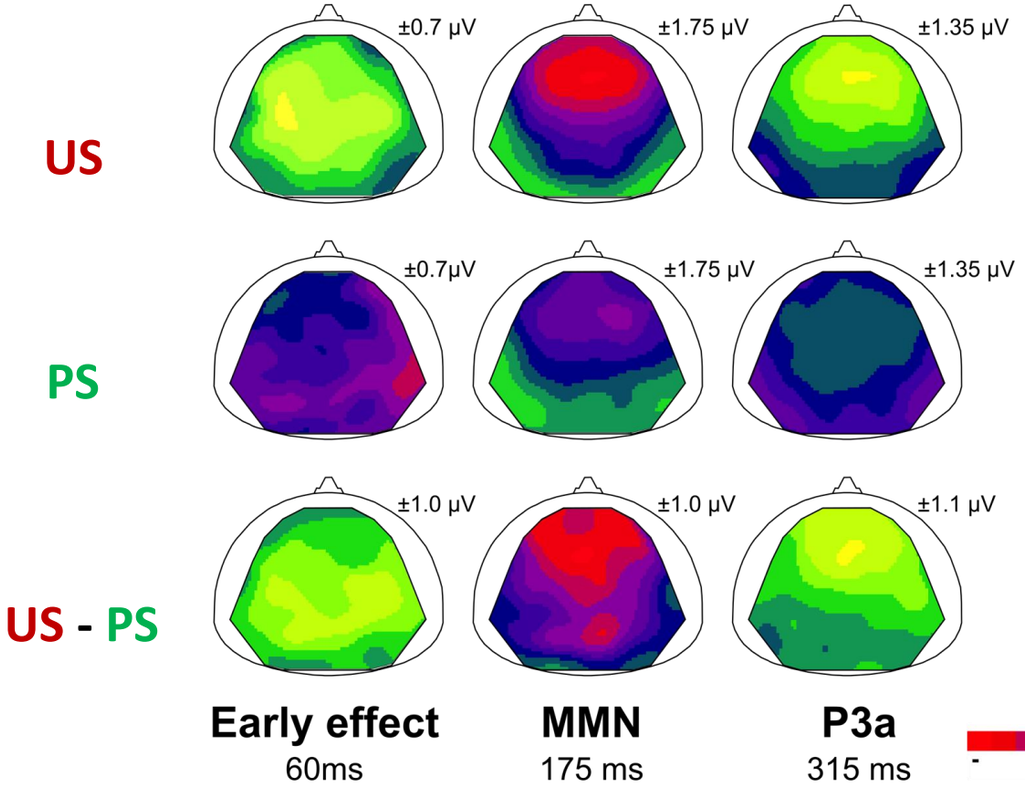
Neurophysiological correlates of implicit perceptual learning

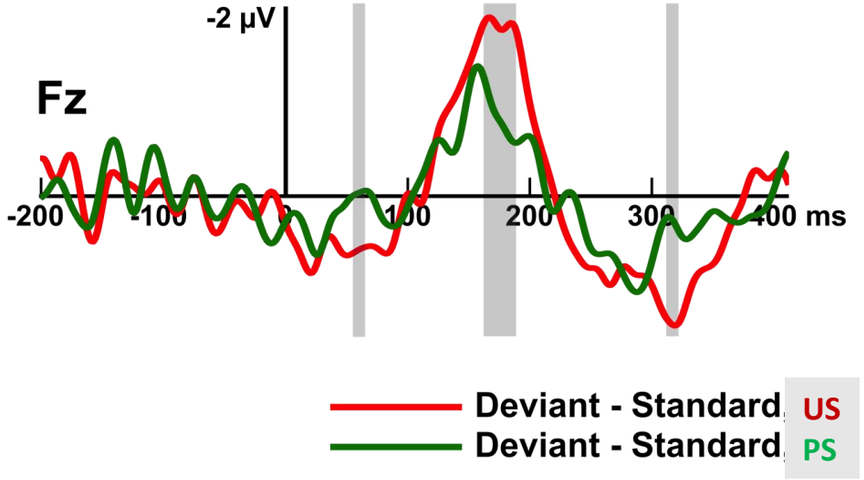
- **Deviance responses should be modulated by predictability**

Effect of predictability

US: Unpredictable sequence
PS: Predictable sequence

Lecaignard et al. 2015





2 – Inferring learning mechanisms from EEG/MEG data

Meta-Bayesian approach

- **Single trial modulations of evoked responses should reflect perceptual learning**

✓ Alternative models

$$\dot{x} = f(x, \theta, u)$$

$$y = g(x, \varphi) + \varepsilon$$

M1 M2 M3 M4 M5	Bayesian learner *with different forgetting parameter value*
M6 M7 M8	Change detection models Null model (noise)

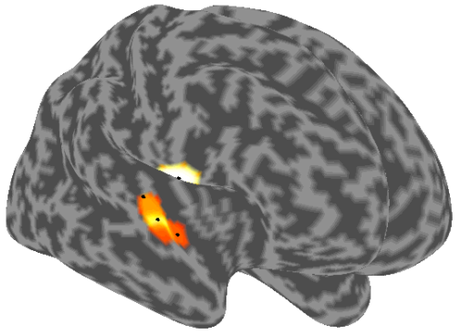
$$p(\mu|m,l) = \frac{\Gamma(m+l)}{\Gamma(m)\Gamma(l)}\mu^m(1-\mu)^l$$

Ostwald et al. 2012

Bayesian Surprise

$$= \mathrm{KL}\left(p\left(\mu|m_{w_{n-1}}, l_{w_{n-1}}\right)\|p\left(\mu|m_{w_n}, l_{w_n}\right)\right)$$

✓ Data features: individual single-trial source activity in clusters estimated from EEG-MEG group inversion



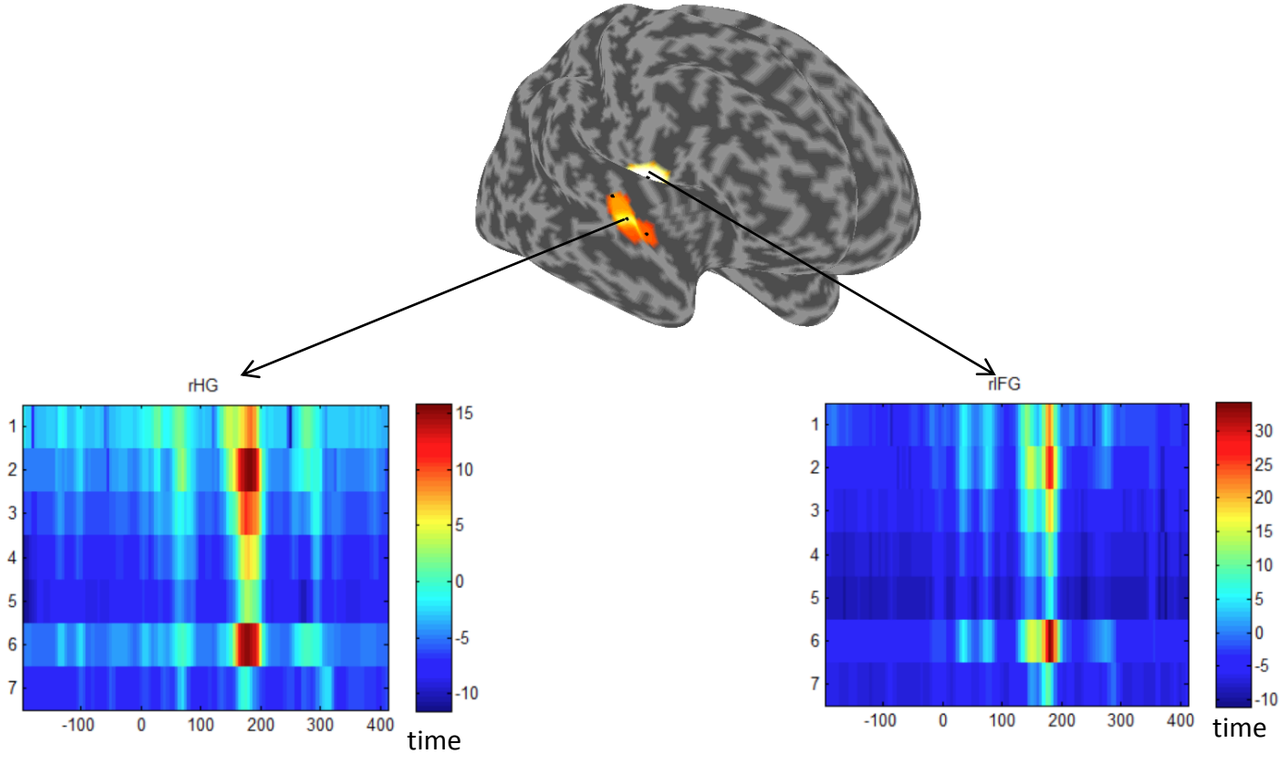
Lecaignard et al. in prep (a)

2 – Inferring learning mechanisms from EEG/MEG data

Preliminary results

- **Single trial modulations of evoked responses should reflect perceptual learning**

✓ Model comparison in a single subject, single block (condition **US**)



Relative Free-energies or Bayes Factors w.r.t null model (M8)

M1 M2 M3 M4 M5	Bayesian learner *with different forgetting parameter value*
M6 M7 M8	Change detection models Null model (noise)

Lecaignard et al. in prep (b)
VBA toolbox, Daunizeau et al. 2014

3 – Online Bayesian inference and design optimization

3 – Online Bayesian inference and design optimization

Motivation for Adaptive Design Optimization

- ✓ Fine neurobiological and/or computational models of evoked responses and their modulations may tell us more about information processing in the brain
- ✓ Given such models, we may be in a position to disentangle between alternative hypothesis about ongoing perceptual inference and learning in a given subject/patient
- ✓ However, what design to choose? How to optimize it for each individual?

Basic idea

- ✓ To make use of our ability to process data in real-time in order to optimize design parameters online, for a given individual, in the aim of selecting the most likely model both accurately and fast

Wald 1947, Cavagnaro et al. 2009, Myung et al. 2013

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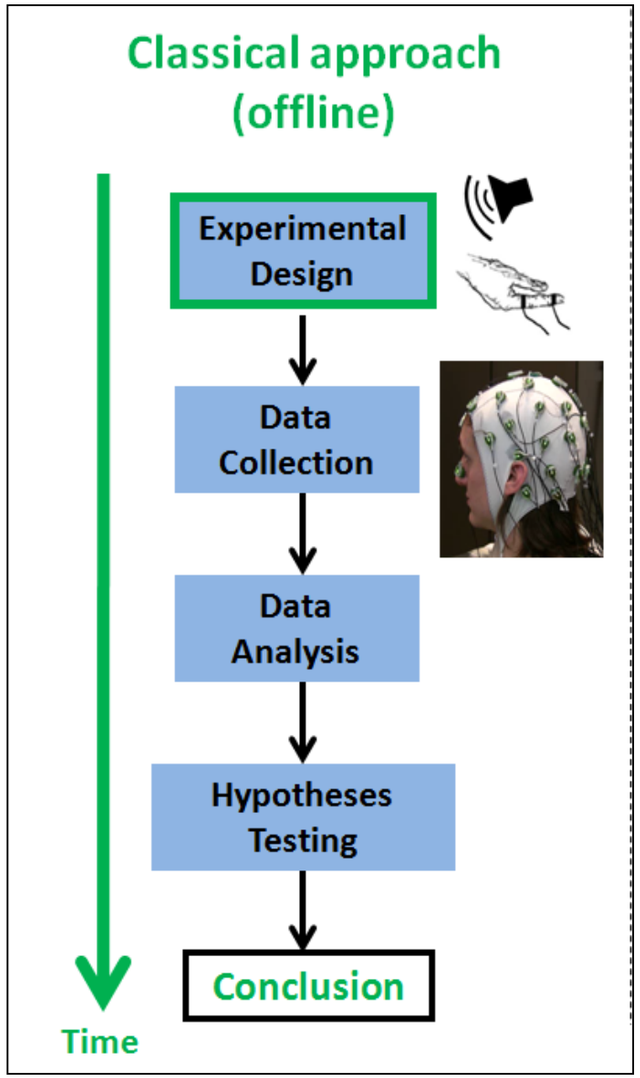
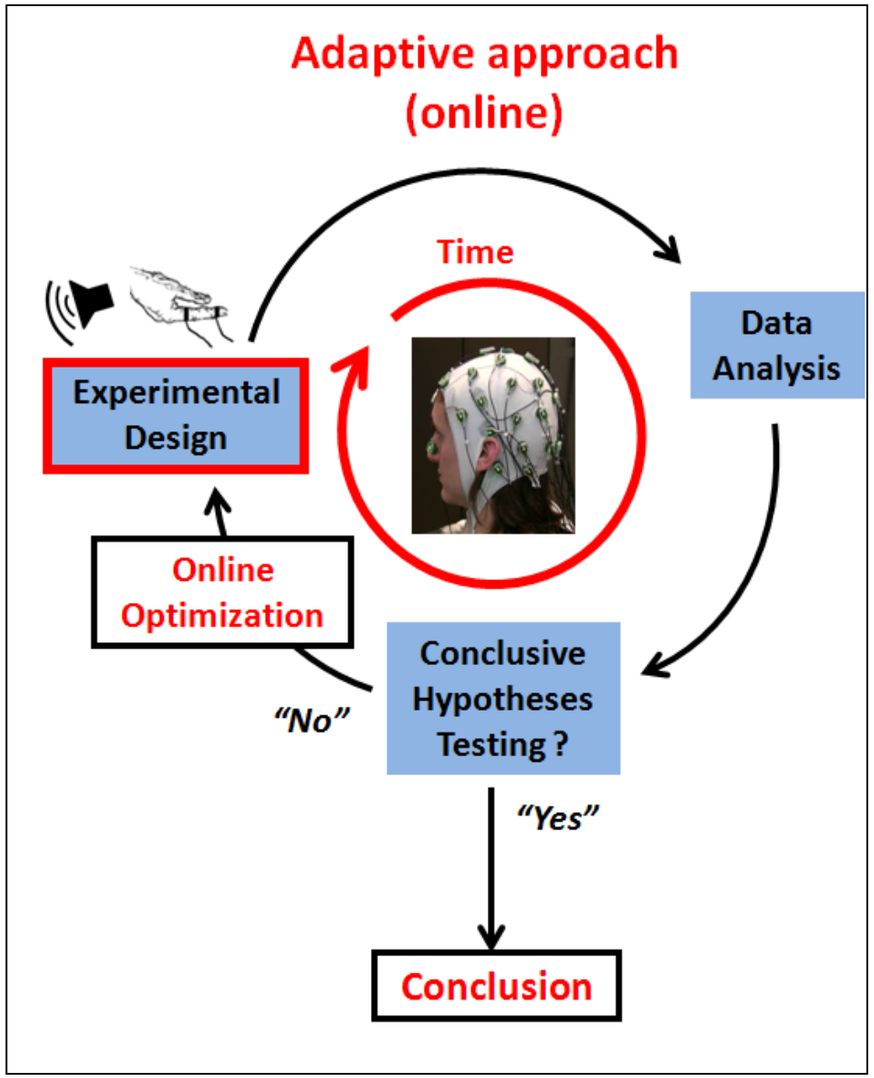
General principle of Adaptive Design Optimization





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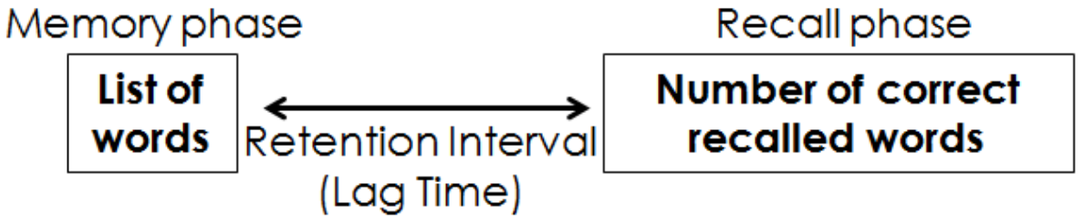
A simple example: behavioural task of word memorization

✓ Experimental context



u (design variable) = Lag time

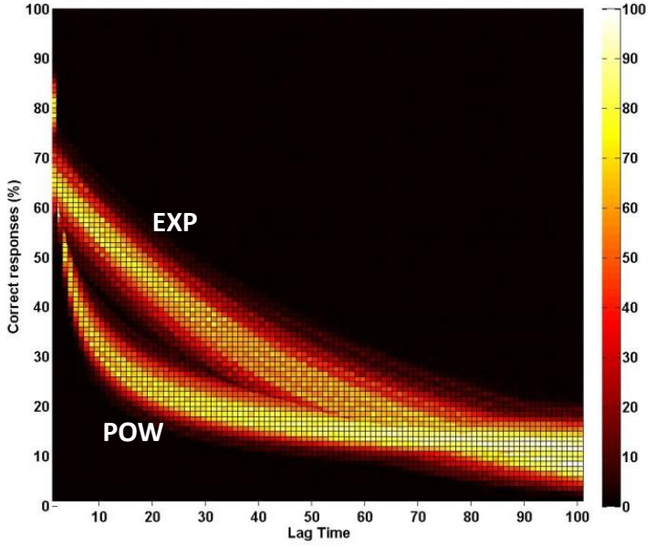
Y (observations) = number of recalled words

✓ Hypothesis

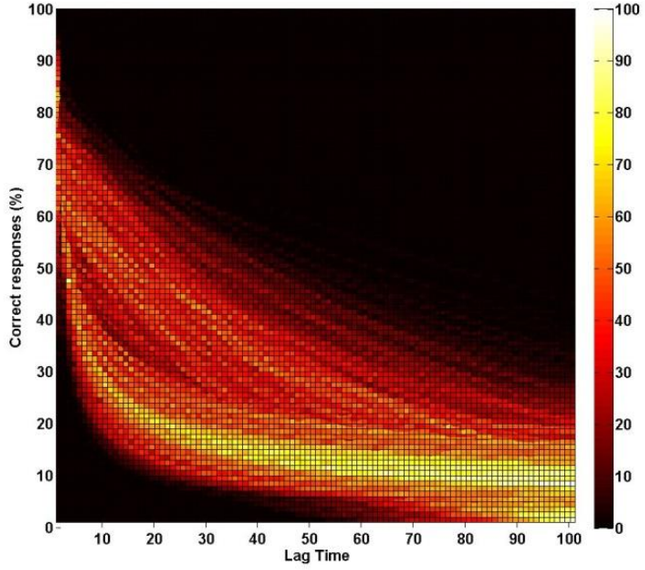
Models of "forgetting"

Model	Equation
Power (POW)	$p = a(t+1)^{-b}$
Exponential (EXP)	$p = ae^{-bt}$

Predictions
(Low uncertainty about model parameters)



Predictions
(High uncertainty about model parameters)



Cavagnaro et al. 2009, Myung et al. 2013

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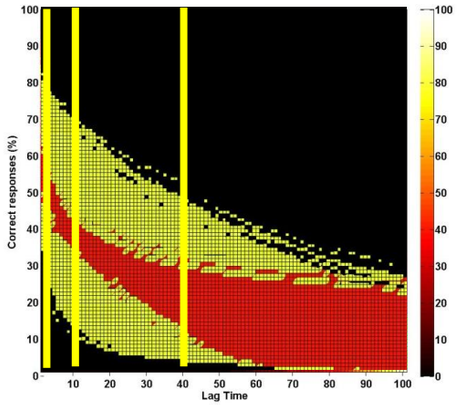
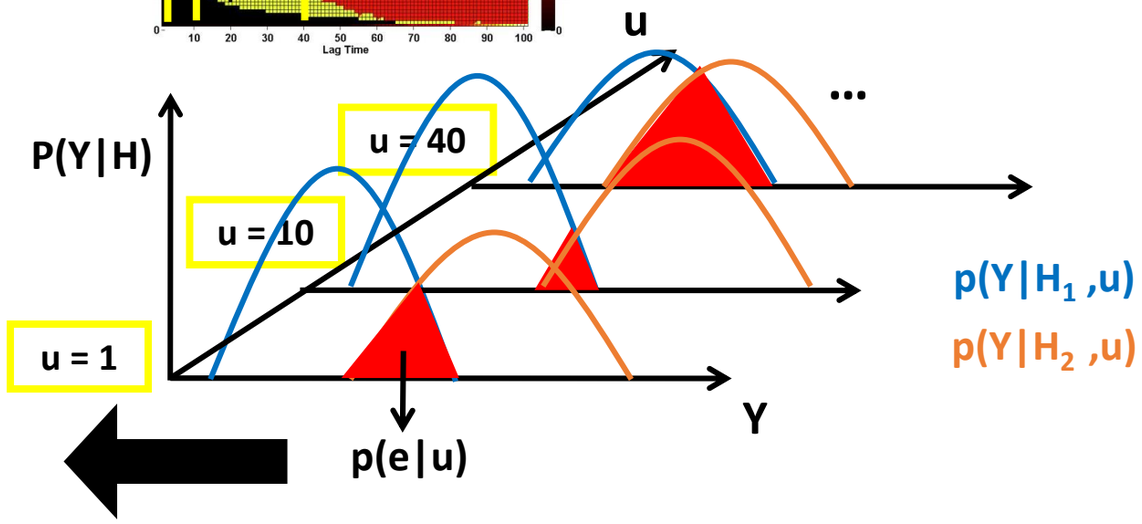
Optimization criterion

✓ **Minimizing the risk of model selection error**



H_1 : EXP
H_2 : POW



$u^* = \text{argmin}\{p(e|u)\}$

Danizeau et al. 2011

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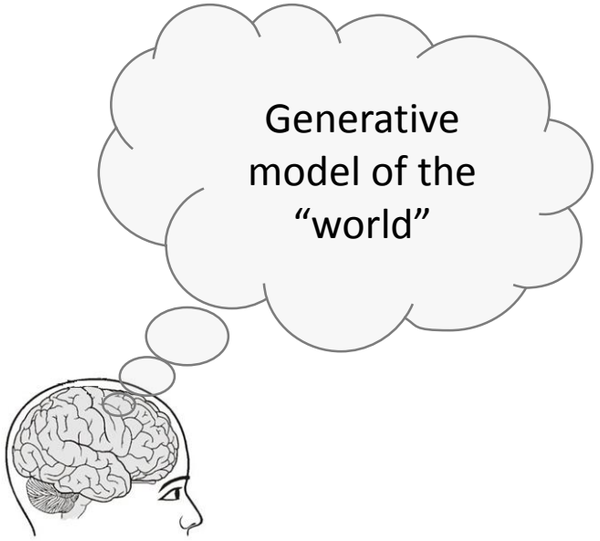
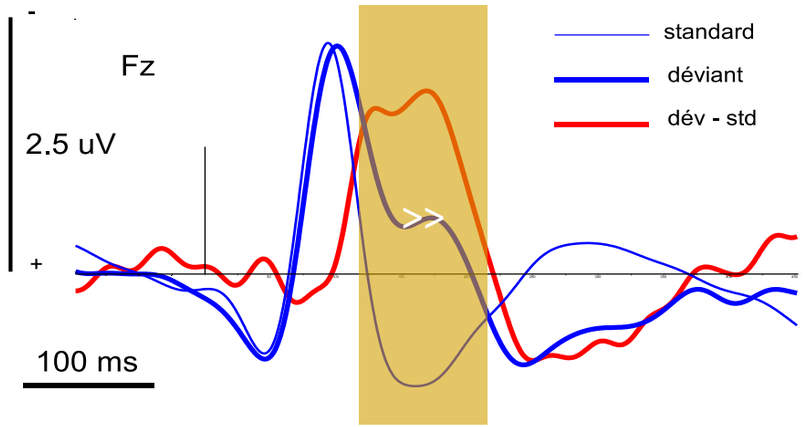
Extension to more complex (DCM-like) models

✓ **Extension to more complex, multidimensional models**



+

ERPs



Evolution function ***f***

$$\dot{x} = f(x, u, \theta_{evol})$$

Observation function ***g***

$$y = g(x, \theta_{obs}) + \varepsilon$$

Danizeau et al. 2011
Sanchez et al. 2014

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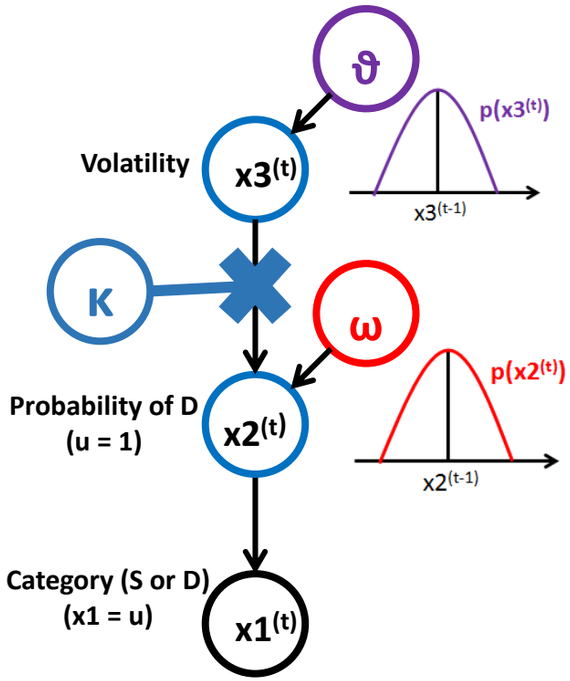
Disentangling between learning models

✓ **A perceptual learning model family: hierarchical Gaussian filters**



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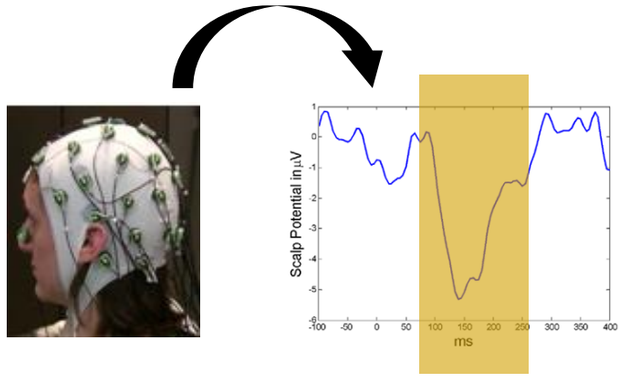

KL-divergence
between prior and post beliefs

Observer function ***g***

$$y = g(x, \theta_{obs}) + \varepsilon$$

Mathis et al. 2011 & 2014

Sanchez et al. 2014

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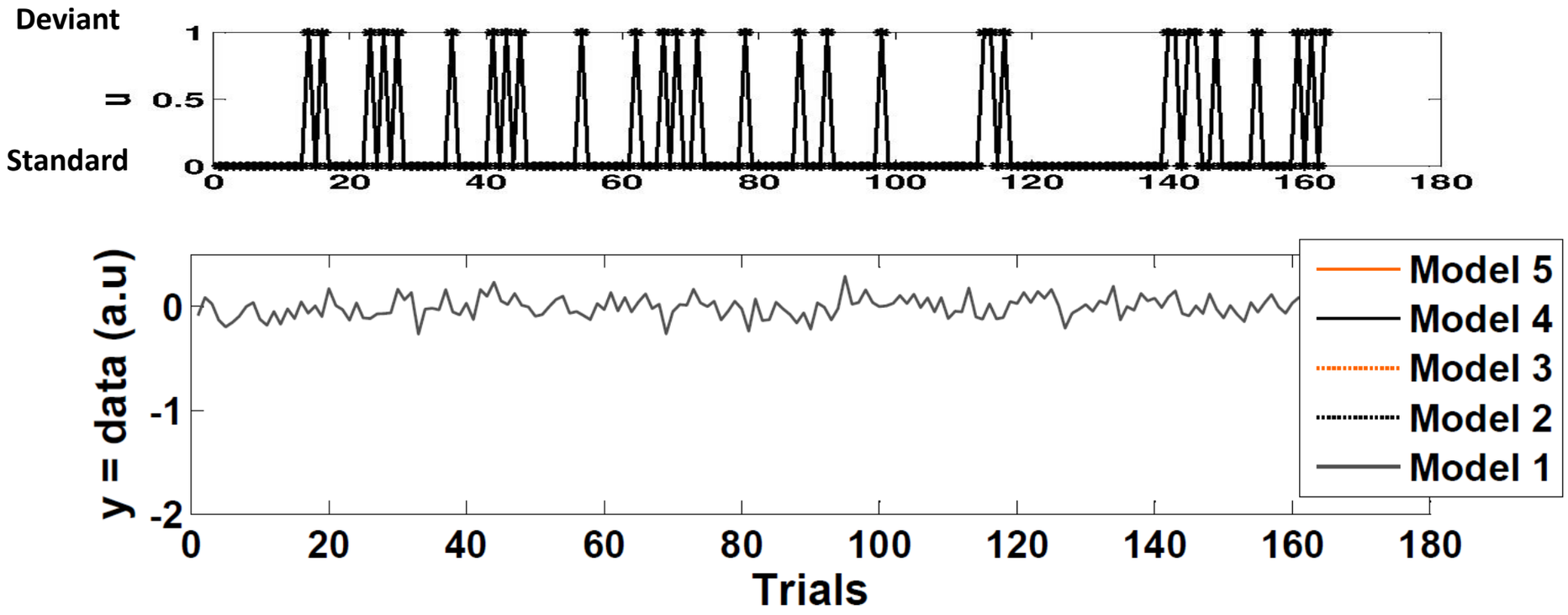
Disentangling between learning models

✓ **Simulations: alternative models and predictions**

Models	ω values	κ values (if κ = 0, no 3rd level)	ϑ values	Ability to track events probabilities	Ability to track environmental volatility
M1	-Inf	0	-	No	No
M2	- 5	0	-	Low learning	No
M3	- 4	0	-	High learning	No
M4	- 5	1	0.2	Low learning	Yes
M5	- 4	1	0.2	High learning	Yes



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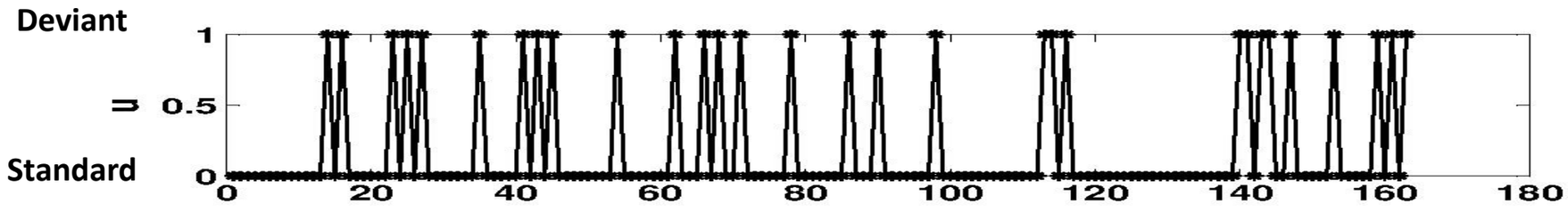

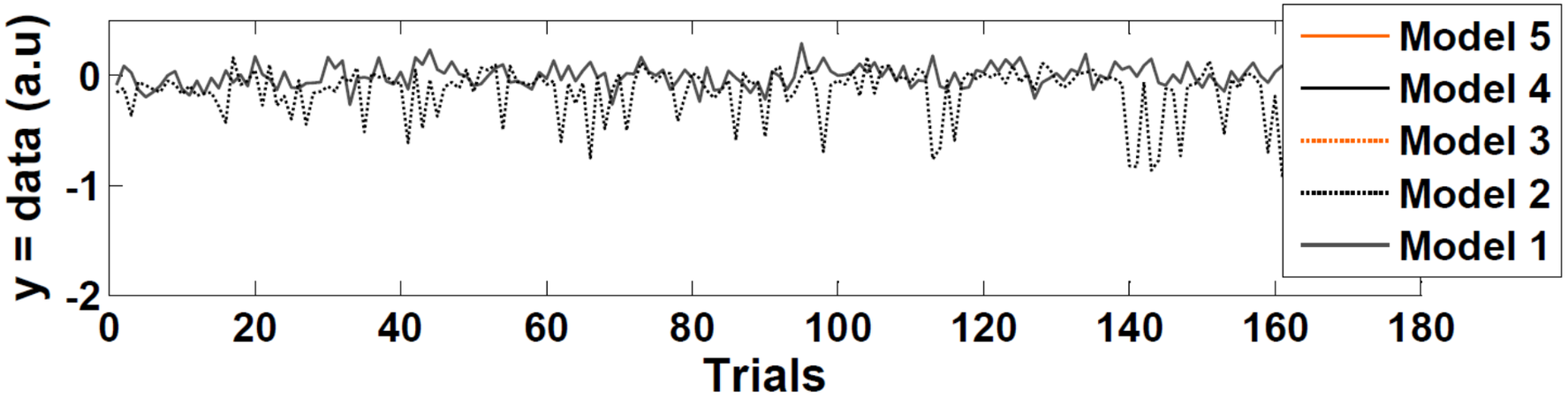

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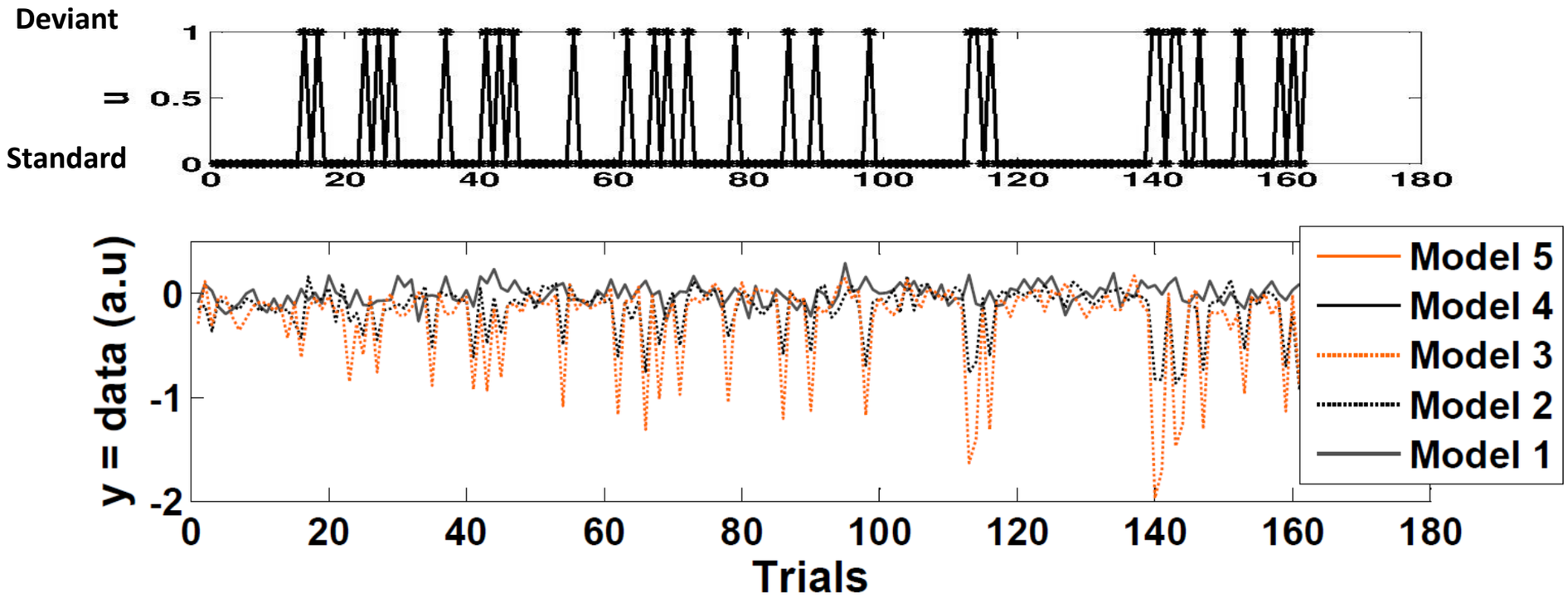

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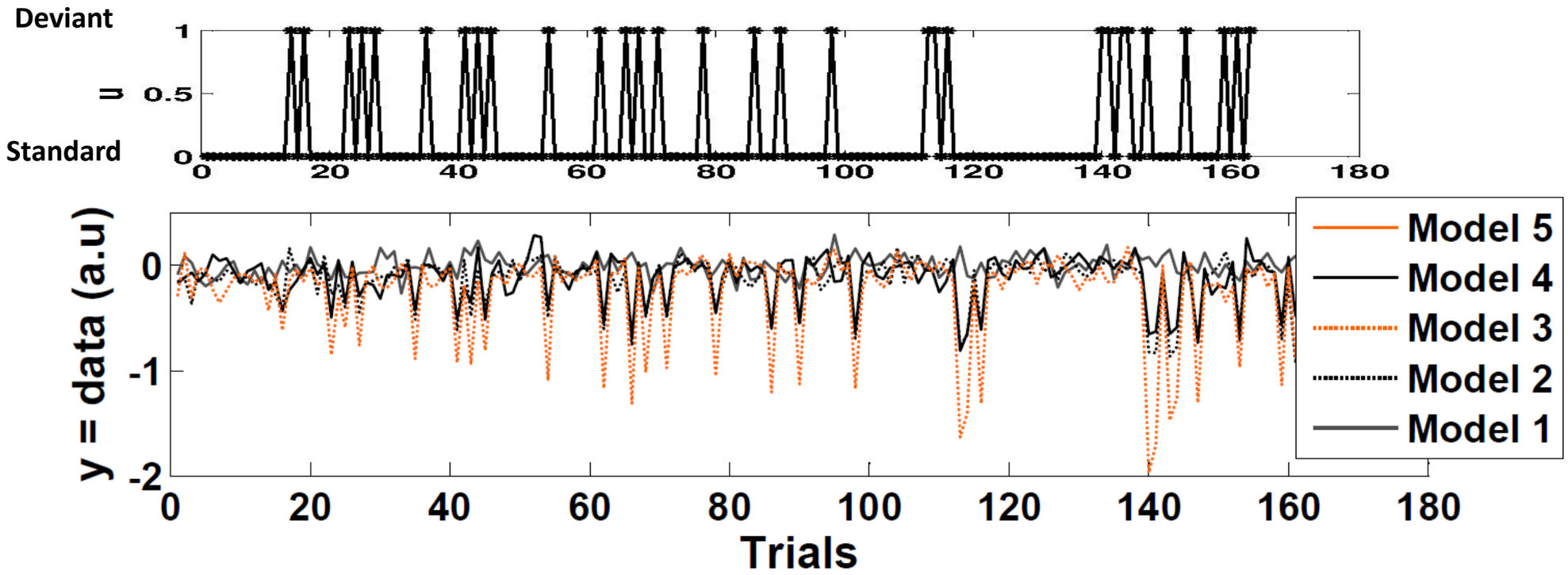

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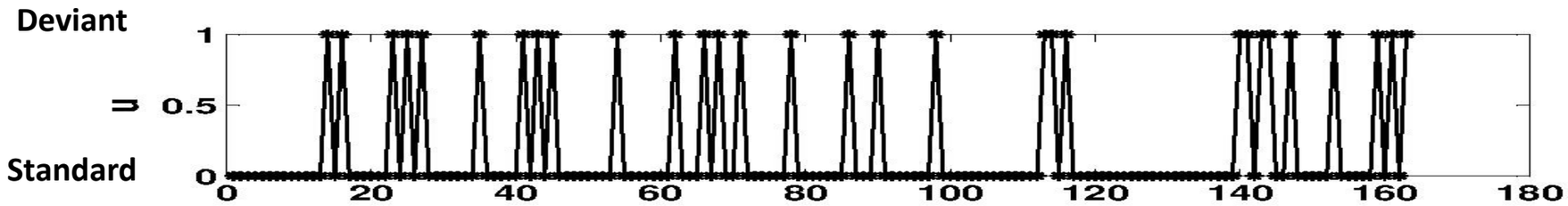

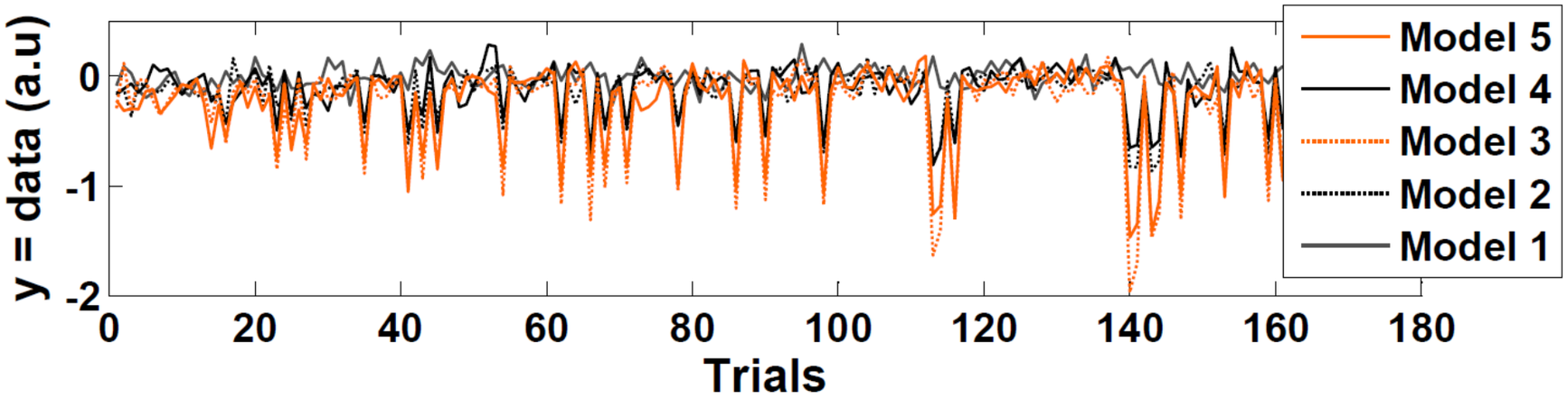

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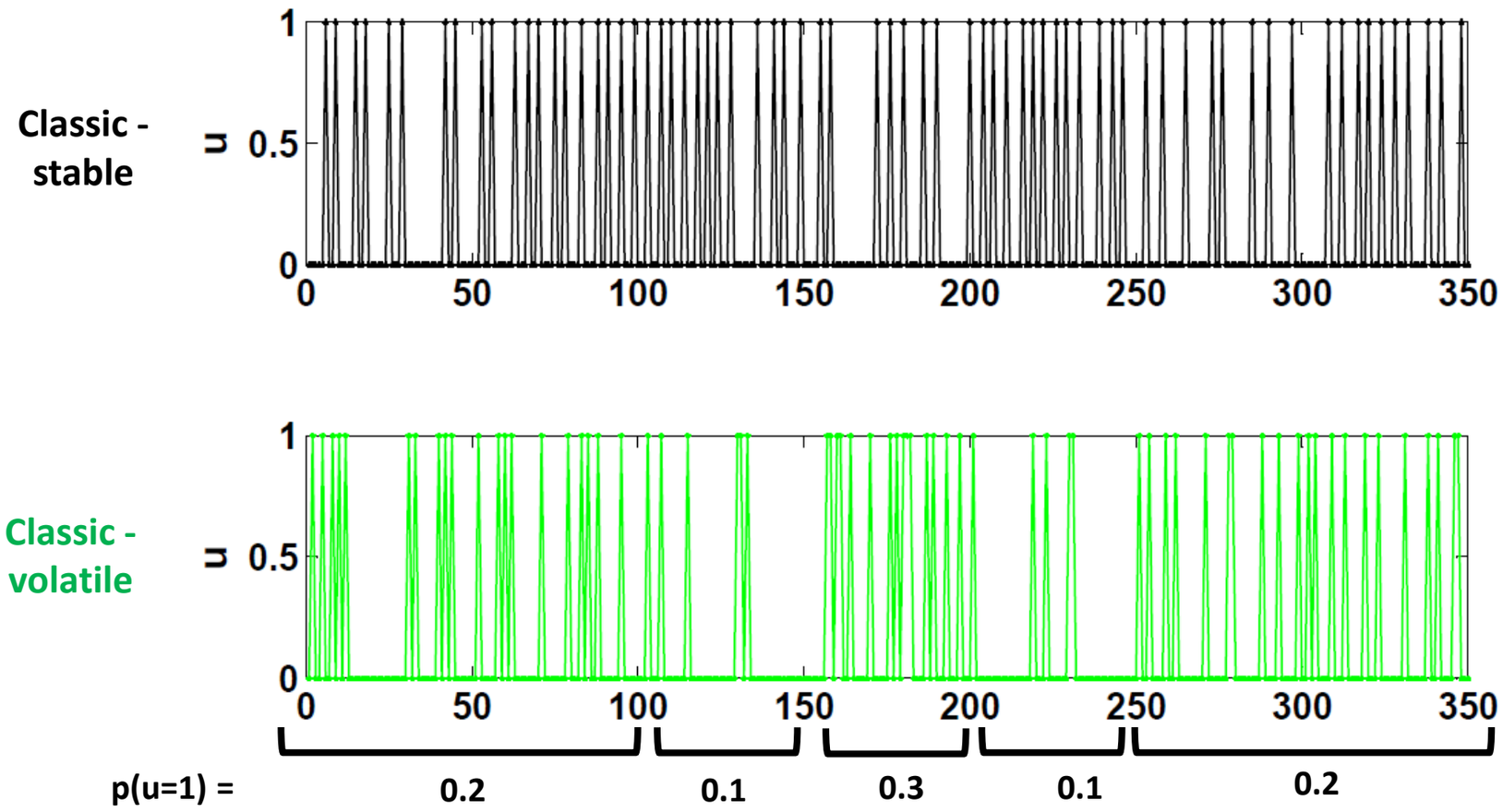
Disentangling between learning models

✓ **Static designs to be compared with ADO**



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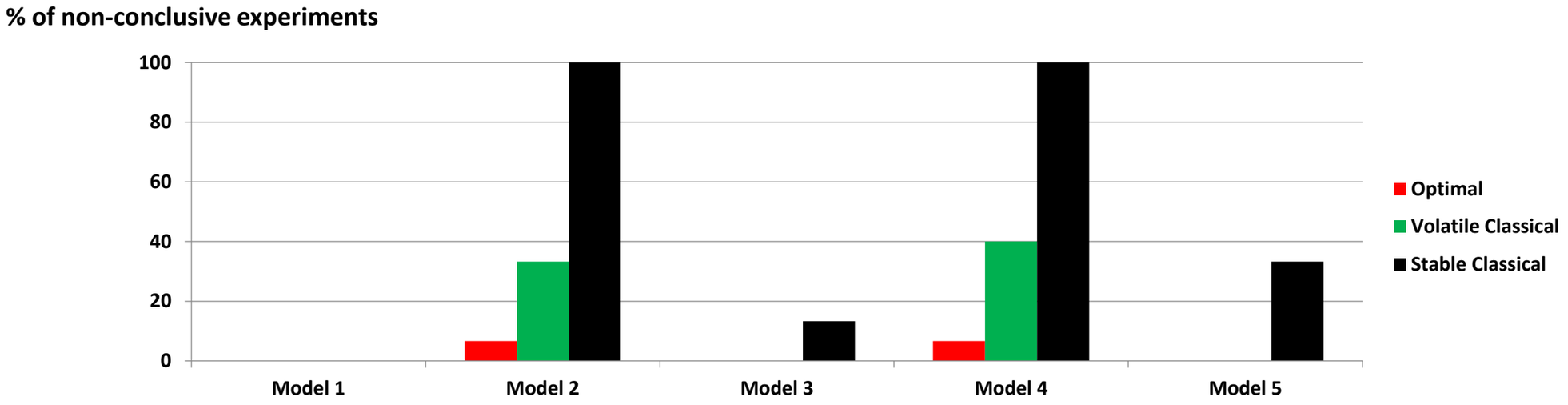
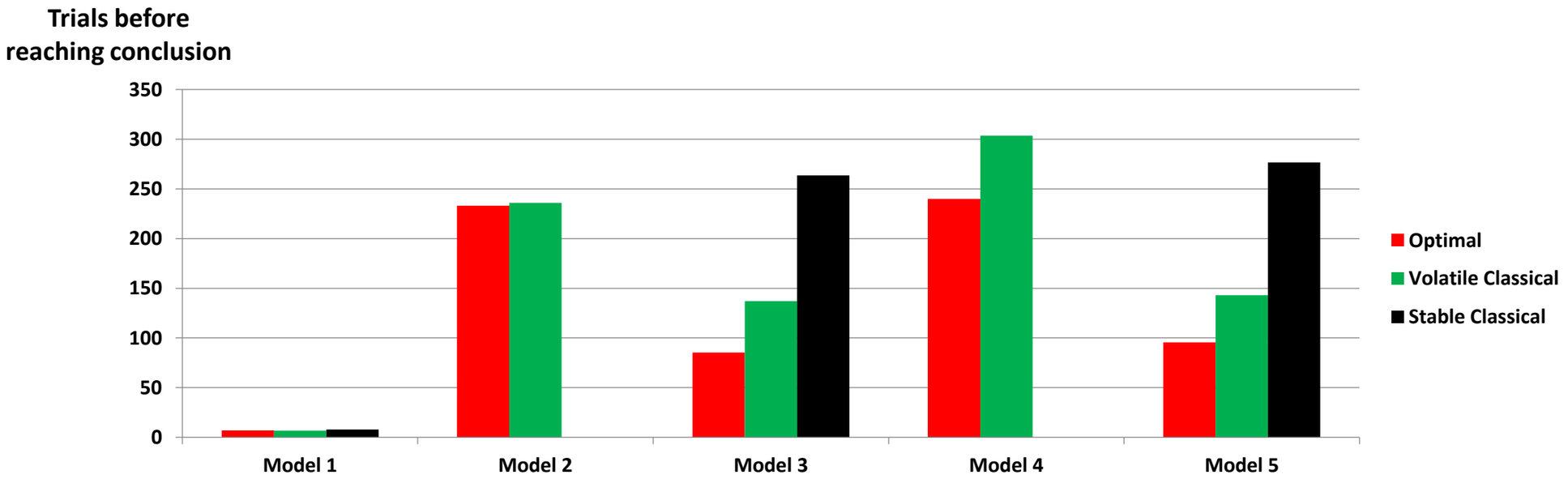
Disentangling between learning models

✓ **Results on Monte Carlo simulations**





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Conclusion

- **1 - EEG/MEG Source reconstruction**
  - Bayesian inference on models and parameters
  - Group level inference
  - EEG/MEG data fusion

- **2 - Inferring learning mechanisms through EEG/MEG responses**
  - meta-Bayesian inference
  - Model comparison

- **3 - Online Bayesian inference and design optimization**

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